Student Using Anticipatory Strategies on Equal Sharing Word Problems

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This story follows the path a group of teachers took in developing and implementing a lesson on fractions. Prior to the lesson, the teachers observed all of the students in one teacher's class completing a related mathematics problem, and their observations provided insight into these students' understanding of fractions through an analysis of the types of strategies her students were using to solve the problem. The teachers decided a worthwhile learning goal for these students is to begin to think in advance about the number of sharers or the amount to be shared (i.e.., anticipatory thinking) when solving equal sharing problems that involving fractional quantities. They developed a lesson designed to help move students toward this goal, implemented the lesson, and reflected on the results.

Relevant Florida Mathematics Standards

MAFS.5.N.2.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Background Information

Consider reviewing chapter one from Extending Children's Mathematics: Fractions and Decimals. This chapter provides a more detailed explanation of equal sharing problems and the strategies that children typically use when solving them.

Empson, S. & Levi, L. (2011). *Extending Children's Mathematics: Fractions and Decimals*. Portsmouth, NH: Heinemann.

Analyzing Student Thinking

The following problem was posed to a classroom of students in late September. The students had access to pencil and paper and were asked to solve the problem in whatever way made sense to them. At this point in the year, the students had not yet been working with fractions in mathematics class. The purpose of this assessment was to learn how these students were thinking about these ideas before the instructional unit started. Learning about students' knowledge before the unit started would help target subsequent instruction to the needs of these individual learners.

> *Mrs. Duffy made a pancake breakfast for her class. There were 10 pancakes left over. Save gave these 10 pancakes to 4 of the kids who were still hungry. She told them they must share the pancakes equally so that each of them got the same amount. How much pancake did each kid get?*

Types of Strategies for Problem A

As the students worked, the teachers considered the following categories as general groups for classifying the strategies the students were using.[1]

A student that uses a *nonvalid, no coordination between sharers and shares* strategy pays attention to only one requirement of an equal sharing situation, either the need for equal shares or the need to use up everything. For the four kids sharing ten pancakes problem, a student might give two pancakes to each person (demonstrating that they understand the need for equal shares) but not consider the two left over pancakes (demonstrating that they aren't paying attention to the requirement that everything should be used up). The reverse could also occur in which a student might distribute the pancakes so that two people get three pancakes each and two people get two pancakes each.

There are two common variations of the *nonanticipatory* sharing strategy. The student understands the requirements of an equal sharing problem (i.e., the need for equal shares and the need to use up everything) and distributes the amount to

1 The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms consider reading Empson, S., & Levi, L. (2011).

This problem would put students in a situation where they would have to decide how to coordinate the number of sharers and the amount to be shared in order to be successful.

be shared among the sharers. Rather than carefully considering how to share the items among the sharers, the student just begins distributing items and working it out as they go along. Two common variations of the *nonanticipatory* sharing strategy follow:

Nonanticipatory, trial and error: For the four kids sharing ten pancakes problem, the student might dividing the pancakes in half and see if they can share everything equally. If that doesn't work, they might repeat this process by dividing pancakes in thirds, in fourths, in fifths, etc. until it works out equally.

Nonanticipatory, repeated halving: Again, for the four kids sharing ten pancakes problem, the student might divide the pancakes in half, then in half again, then in half again until they can share everything equally.

A student using an *additive coordination, sharing one item at a time* strategy understands the requirements of an equal sharing problem (i.e., the need for equal shares and the need to use up everything). The student anticipates in advance how to divide items by coordinating the amount to be shared among the sharers. For the four kids sharing ten pancakes problem, the student might divide each of the ten pancakes into fourths, giving each person one fourth of each pancakes. The student would then use addition to determine that 1/4 + 1/4 + 1/4 + 1/4 + 1/4 + 1/4 + 1/4 + 1/4 + 1/4 + 1/4 = 10/4, or they would count them as 1/4, 2/4, 3/4….10/4. Another possibility is that a student might give each person two pancakes and then divide each of the leftover pancakes into fourths. In both examples, the student has carefully coordinated the amount to be shared among the sharers.

A student using a *multiplicative coordination* strategy coordinates the amount to be shared among the sharers. The student no longer needs to create a concrete model of each item to be shared. Because they understand that one item shared with *n* people results in 1/n pieces for each sharer, they can use multiplication to determine how multiple items would be shared. For the four kids sharing ten pancakes problem, the student might say that each person gets ten fourths because they get one fourth of each of the ten pancakes. Another possibility is that the student immediately responds ten fourths, because they understand connections among fractions, division. In this case, ten is divided by four, which results in the fraction 10/4.

Strategies Used by Students in This Class

After the students solved the problem, the teachers sorted each student's work into the categories described in the previous subsection. Of the fifteen students who completed the problem, 9 of them (60%) used a *sharing one item* at a time strategy, 4 of them (27%) used a *nonanticipatory* strategy, and

2 of them (13%) did not demonstrate a valid strategy. No students were observed using a *sharing groups of items* strategy or a *multiplicative coordination* strategy.

Learning Goal for This Group of Students on This Day

Because 40% of the class was using a *nonvalid* or *nonanticipatory* strategy, the teachers developed the following learning goal for a lesson they would teach these students later that day.

> *Students will be able to use an* ***additive coordination*** *strategy to solve equal sharing problems.*

Planning for the Lesson

The teachers worked to develop a problem for students to solve that would still enable those students using *nonvalid* and *nonanticipatory* strategies to be successful in solving the problem while at the same time providing opportunities for students to further develop or begin using *additive coordination*, *sharing one item at a time* strategies to solve the problem. Some students were already using these more sophisticated strategies, and they can share their ideas with their peers to promote learning for all students. The teachers decided the problem should involve a scenario where the number of items to be shared was one greater than the number of sharers. Additionally, creating a problem in which there were four sharers would allow for students using a *nonvalid* strategy to easily cut items in one of the most intuitive way that students tend to cut (i.e., halves, then halves again). The teachers were aware that these numbers allowed students using a *nonanticipatory*, *repeated halving* strategy to be successful without having to reason through why dividing items in fourths (i.e., half and half again) works so well. However, the teachers decided that they wanted to press on this further in the lesson. The context of clay was used, because it allowed students to envision a material that was commonly shared and easily malleable.

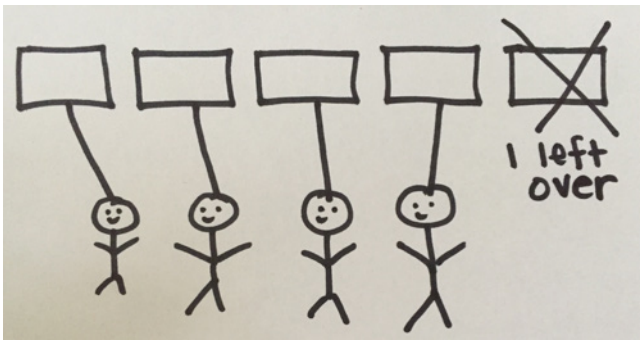

Figure 1. A ***nonvalid, no coordination between sharers and shares*** *strategy with not all the clay being shared.*

The teachers ultimately decided on the following problem:

> *Four students want to share five bars of clay equally. How much clay will each person get if they use up all of the clay?*

The teachers also noted that some of the students had solved the Ms. Duffy problem that morning by giving two pancakes to each person and ignoring the remaining two pancakes. They decided it will be very important to provide scaffolding to help students make sense of the problem and know that they must share all fived of the bars of clay among the students and make sure each student gets the same amount.

The teachers anticipated the following student strategies.

Nonvalid, no coordination between sharers and shares: Student would give each person a whole bar of clay and just indicate that there is one bar of clay left over (see Figure 1).

Nonvalid, no coordination between sharers and shares: The student will give one whole bar of clay to each person and will halve the one remaining bar of clay, consequently not providing equal shares (see Figure 2).

Nonanticipatory, trial and error: The student would try dividing the five bars of clay into fraction amounts and sharing them, until the came across a division of clay bars that would share equally

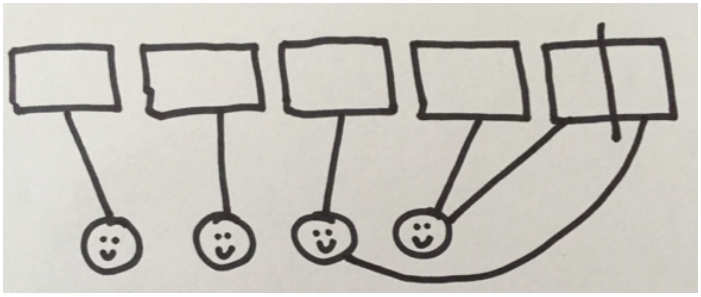

Figure 2. A ***nonvalid, no coordination between sharers and shares*** *strategy without equal shares.*

among four. For example, a student would try cutting the five bars of clay into different amounts (halves, then thirds, then fourths) to determine a fractional quantity that could be shared equally among all the sharers. (See Figure 3.)

Give one to each sharer and divide the amount left over between the number of sharers: The student will give one whole bar of clay to each of the four sharers and then divide the fifth bar of clay into four equal parts to give each child 1/4 of a bar of clay (see Figure 4).

Pre-selecting Strategies for Students to Share

Prior to the lesson, the teachers preselected two student strategies related to the learning goal. These strategies were to be shared in the introductory phase of the lesson. First, they identified a student who had a *nonvalid, no coordination between sharers and shares* strategy, giving one bar of clay to each sharer and then incorrectly divide the left over into halves. This would highlight that one requirement of an equal sharing situation wasn't met: the need for equal shares. The second strategy they identified was a student who gave one bar of clay to each sharer and then dividing the remaining bar by the number of sharers. A student who did this with anticipatory thinking (ie. a student who used an *additive coordination, sharing one item at a time* strategy) could explain clearly a solid reason for dividing the left over in fourths. This exposure would benefit the students who used a *nonanticipatory, trial and error*, or *nonanticipatory, repeated halving* strategy, by helping them see how other students are thinking about the equal shares required in this problem.

Lesson Plan

The following lesson plan was implemented after the initial information was collected on the students. In the planning for this lesson, the teachers established the following learning goal:

> *Students will be able to use an* ***additive coordination*** *strategy to solve equal sharing problems.*

Before starting the lesson, carefully anticipate the two types of student responses that will be shared in the closing.

1. Be sure that students have access to paper and pencil while they work on the problem that will be presented in the lesson. Consider also offering students colored pencils, if they find it would help them to clearly communicate how they solved the problem.

Figure 3. A valid strategy with ***nonanticipatory*** *thinking.*

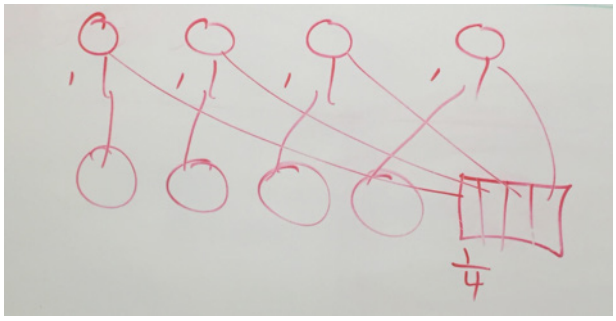

Figure 4. An ***additive coordination, sharing one item at a time*** *strategy.*

2. Read the following problem to the students: *Four students want to share five bars of clay equally. How much clay will each person get if they use up all of the clay?*

3. Take some time to make sure students understand the problem before they are asked to solve it. To support students in making sense of the problem itself, consider asking the following sequence of questions.

   i. "What do you know about clay?" If students do not bring up how clay can be molded and cut, ask a question such as, "Can you share clay among people? What is that like to cut clay?"

   ii. "How many people are going to share the clay?" Follow up with students who may have missed this important detail.

   iii. "How much clay do we have?" Again, follow up with students who may have missed this important detail.

   iv. Take a moment to encourage students to take a big-picture view of the problem. "Is that going to be enough clay for everybody to get some? How do you know? Will there be any clay left over?" Consider inviting a student to start forming a pictorial representation of the context.

   v. "What does the problem say has to happen with the clay?" Make sure that the students understand that there are four students and five bars of clay and that all the clay must be shared equally with no left over clay.

4. Remind students it is important that they record their thinking so that someone else can clearly see how they arrived at their answers.

5. Invite students to solve the problem on their own. Tell students if they finish the problem before the rest of their classmates that they can work on finding a different way to solve the problem.

Some students noticed that there were two answers to the problem (i.e., 5/4, 1 ¼), and they were perplexed by this. The teachers noted this could create a great opportunity on another day to explore ways to express fractions greater than one and to have students justify how 4 fourths was equivalent to 1 whole.

6. Circulate to observe how students are approaching the problem.

7. Look for students who are using the pre-selected strategies (i.e., a *nonvalid, no coordination between sharers and shares* strategy involving students assigning one bar of clay to each sharer and dividing the left over into halves, an *additive coordination, sharing one item at a time* strategy involving the student assigning one bar of clay to each sharer and then dividing the remaining bar by four, the number of sharers).

8. Once you identify students who are using the pre-selected strategies and clearly communicate their thinking, ask them if they would be willing to share their thinking during the summarize portion of the lesson. This allows those students to begin thinking about what they would like to say in reference to their work when they share it with the class.

9. When you find students who have correctly finished the task, consider asking them some of the following questions:

    i. "Has all of the clay been shared?"

    ii. "Does each person get the same amount of clay?"

    iii. "Can you show me the clay that one person gets?"

    iv. "How else can you represent your answer?" "How does the amount of clay that each student gets relate to the clay shared and the number of people sharing it?"

10. If you find students that have finished early, and solved the problem in more than one way, consider providing them with another problem.

    i. For the students using a *nonvalid, no coordination between sharers and shares* strategy, it might be best to keep the number of sharers as two or four, because this would allow for a greater chance for success for these students.

    ii. For the students using a *nonanticipatory, repeated halving* strategy, a problem like six students sharing five bars of clay would require them to more carefully consider coordinating the number of sharers with the amount to be shared because their repeated halving strategy won't work.

    iii. Six students sharing five bars of clay would also be an appropriate problem for the students using a *nonanticipatory, trial and error* strategy or an *additive coordination, sharing one item at a time* strategy.

    iv. To challenge the students using an *additive coordination, sharing one item at a time* strategy, ask students if they might be able to solve this problem mentally (without drawing a model of the clay and the people).

11. Invite two to three students to share their thinking, beginning with a student who had a *nonvalid, no coordination between sharers and shares* strategy, giving one bar of clay to each sharer and then incorrectly divide the left over into halves was beneficial to highlight that one requirement of an equal sharing situation wasn't met: the need for equal shares.

12. After eliciting comments from the class about this strategy, a second strategy to share would be a student who gave one bar of clay to each sharer and then divided the remaining bar by the number of sharers. During the discussion of this strategy, ask the class questions about why the left over bar of clay was cut into fourths would highlight that the number of sharers can be coordinated to the number of items being shared.

Reflection

The lesson was implemented as described in the previous section. There are not many artifacts from the lesson available to display here, but the teachers' observations and discussions are noted in the following paragraphs.

Students who were not considering both requirements of an equal sharing situation became more aware that the way they shared the clay wasn't "fair." Some students seemed surprised that other students seemed to immediately know which way of cutting the clay would work. These latter students—who were using anticipatory thinking—explained their thinking to the other students. The teachers noticed that when students shared their thinking with the class, the other students were more willing to try these strategies. During the wrap up discussion, many students stated that they were going to try that strategy the next time they had a problem like this. It will be interesting to observe whether those students are able to begin using anticipatory strategies on their own in future problems and lessons.

Many students were able to solve the problem and explain how much clay each person would get. While they were able to solve the problem, many of them did not express their answers as a fraction with numerals. The teachers agreed that further emphasis needs to be placed on teaching these children how to express fractions using conventions of written notation.

An unexpected debate occurred in the class discussion. Some students noticed that there were two answers to the problem (i.e., 5/4, 1 ¼), and they were perplexed by this. The teachers noted this could create a great opportunity on another day to explore ways to express fractions greater than one and to have students justify how 4 fourths was equivalent to 1 whole.

In addition to exploring 5/4 and 1 ¼ and continuing to model fraction notation, the teachers decided that important next steps in instruction for this class will be to pose more equal sharing problems which cannot be solved by repeated halving. The problem (i.e., six students sharing 5 bars of clay) from this lesson that was given to the students who finished the first problem early is a good candidate problem for all of the students to solve. This problem would put students in a situation where they would have to decide how to coordinate the number of sharers and the amount to be shared in order to be successful.

The teachers also thought it would be a good idea to orchestrate more discussions that would address learning goals for the students who are already using an *additive coordination, sharing one item at a time* strategy. They would really like to see these students divide each item among the number of sharers so that they can more readily begin moving toward a *multiplicative coordination* strategy.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**